

Minimal parameterizations for modified gravity

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The increasing precision of cosmological data provides us with an opportunity to test general relativity (GR) on the largest accessible scales. Parameterizing modified gravity models facilitates the systematic testing of the predictions of GR, and gives a framework for detecting possible deviations from it. Several different parameterizations have already been suggested, some linked to classifications of theories, and others more empirically motivated. Here we describe a particular new approach which casts modifications to gravity through two free functions of time and scale, which are directly linked to the field equations, but also easy to confront with observational data. We compare our approach with other existing methods of parameterizing modified gravity, specifically the parameterized post-Friedmann approach and the older method using the parameter set $\{\mu, \gamma\}$.

We explain the connection between our parameters and the physics that is most important for generating cosmic microwave background anisotropies. Some qualitative features of this new parameterization, and therefore modifications to the gravitational equations of motion, are illustrated in a toy model, where the two functions are simply assumed to be constant parameters.

1. INTRODUCTION

General Relativity (GR) has been confronted with many theoretical and experimental tests since its birth in 1915. From the gravitational lensing experiments in 1919 [1] up to the extensive studies and tests of the theory in the 1960s and 1970s [2–6], the theory has been confirmed observationally and theoretically bolstered in many different respects. Weak field gravity on experimentally accessible scales has been so well tested that there are only two remaining directions in which we might find modifications to GR: strong field gravity which may be probed by studying black holes; and gravity at large scales and early times, which is the cosmological arena.

Cosmology has challenged GR with two, yet to be fully understood discoveries: dark matter and dark energy [7, 8]. Along with these two phenomena, the lack of renormalizability in GR [9] and the apparently exponential expansion in the very early Universe [10] are usually taken as signs for the incompleteness of the theory at high energies. Due to these shortcomings in GR the study of modified gravity has become a broad field. Scalar-tensor theories [11, 12], $f(R)$ theory [13, 14], Horava-Lifshitz theory [15], multidimensional theories of gravity [16, 17], and many other suggestions have been made in the hope of finding, or at least deriving some hints for, a fully consistent theory that can successfully explain the observations and satisfy the theoretical expectations. (Ref. [9] has an extensive review).

The new data coming from various experiments such as the *WMAP* and *Planck* satellite measurements of the cosmic microwave background (CMB) anisotropies [18],

and the WiggleZ [19] or Baryon Oscillation Spectroscopic Survey [20] measurements of the matter power spectrum, provide us with an opportunity to test specific modified theories of gravity. However, since there are many different modified theories, all with their own sets of parameters, there has recently been some effort to come up with a way to describe generic modified theories using only a few parameters, and to try to constrain those parameters with general theoretical arguments and by direct comparison with cosmological data.

Parametrizing modified theories of gravity with a small number of parameters has the benefit of tracking the effects of modified gravity on a number of different observables consistently and systematically, rather than considering the consistency of each single observable with GR predictions. This can, at least in principle, lead to the acceptance, rejection, or constraining the theory space of modified gravity models.

The parametrized post-Friedmann (PPF) approach, as described in Ref. [21], is an effective way to parameterize many of the modified theories of gravity. However, it is not really feasible to constrain its more or less dozen additional parameters even with the power of Markov Chain codes such as CosmoMC [22]; there are just too many degrees of freedom to provide useful constraints in the general case. In this paper we will describe a slightly different way to parametrize modified theories of gravity in which we try to retain only a small number of parameters which we then constrain using *WMAP* 9-year [23] and SPT data [24].

In the next section we will describe the formulation of this new parametrization, and will show its connection with PPF and other approaches in Sec. 3. In Sec. 4 we will discuss the results of a numerical analysis using CAMB [25] and CosmoMC, and we will conclude the paper in Sec. 5 with a brief discussion.

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2. MODIFIED GRAVITY FORMULATION

There are two common strategies for modifying gravity. One can start from the point of view of the Lagrangian or from the equations of motion. The Lagrangian seems like the more obvious path for writing down specific new theories, where one imagines retaining some desired symmetries while breaking some others. However, the equations of motion provide an easier way in practice to parametrize a general theory of modified gravity, especially in the case of first order perturbations in a cosmological context.

The evolution of the cosmological background has been well tested at different redshift slices, specifically at Big Bang nucleosynthesis and at recombination through the CMB anisotropies. It therefore seems reasonable to assume that the background evolution is not affected by the gravity modification, with the only background level effect being a possible explanation for a fluid behaving like dark energy.

The linearized and modified equations of motion for gravity can be written in the following form in a covariant theory:

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} + \delta U_{\mu\nu}. \quad (1)$$

Here, $\delta G_{\mu\nu}$ is the perturbed Einstein tensor around a background metric, $\delta T_{\mu\nu}$ is the first order perturbation in the energy-momentum tensor and $\delta U_{\mu\nu}$ is the modification tensor source from any term that is not already embedded in GR.

Since we will be using CAMB and CosmoMC for numerical calculations, we will choose the synchronous gauge from now on, and focus only on the spin-0 (scalar) perturbations. This will make it much more straightforward to adapt the relevant perturbed Boltzmann equations. The metric in the synchronous gauge is written as

$$\begin{aligned} ds^2 &= a^2(\tau) [-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j], \\ h_{ij} &= \int d^3k e^{i\vec{k}\cdot\vec{x}} \{ \hat{k}_i \hat{k}_j h(\vec{k}, \tau) + (\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij})6\eta(\vec{k}, \tau) \}, \\ \vec{k} &= k\hat{k}, \end{aligned} \quad (2)$$

where \vec{k} is the wave vector. Putting this metric into Eq. 1 results in the following four equations [26]:

$$k^2\eta - \frac{1}{2}\frac{\dot{a}}{a}\dot{h} = -4\pi G a^2\delta\rho + k^2 A(k, \tau); \quad (3)$$

$$k\dot{\eta} = 4\pi G a^2(\bar{\rho} + \bar{p})V + k^2 B(k, \tau); \quad (4)$$

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} - 2k^2\eta = -24\pi G a^2(\delta P) + k^2 C(k, \tau); \quad (5)$$

$$\begin{aligned} \ddot{h} + 6\dot{\eta} + 2\frac{\dot{a}}{a}(\dot{h} + 6\dot{\eta}) - 2k^2\eta &= -24\pi G a^2(\bar{\rho} + \bar{p})\Sigma \\ &+ k^2 D(k, \tau). \end{aligned} \quad (6)$$

Here we have used the following definitions:

$$\begin{aligned} \delta T_0^0 &= -\delta\rho & , & & a^2\delta U_0^0 &= k^2 A(k, \tau), \\ \delta T_i^0 &= (\bar{\rho} + \bar{p})V_i & , & & a^2\delta U_i^0 &= k^2 B(k, \tau), \\ \delta T_i^i &= 3\delta P & , & & a^2\delta U_i^i &= k^2 C(k, \tau), \\ D_{ij}\delta T^{ij} &= (\bar{\rho} + \bar{p})\Sigma & , & & a^2 D_{ij}\delta U^{ij} &= k^2 D(k, \tau), \end{aligned} \quad (7)$$

with $\bar{\rho}$ and \bar{p} being the background energy density and pressure, respectively, and a dot representing a derivative with respect to τ . The factors of k are chosen to make the modifying parameters, $\{A, B, C, D\}$, dimensionless.

D_{ij} is defined as $\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij}$.

In general, Einstein's equations provide six independent equations. For the case of first order perturbations in cosmology, two of these six equations are used for the two spin-2 (tensor) degrees of freedom, two of the equations are used for the spin-1 (vector) variables and only two independent equations are left for the spin-0 (scalar) degrees of freedom. This means that Eqs. 3 to 6 are not independent and one has to impose two consistency relations on this set of four equations. These consistency relations are of course the energy-momentum conservation equation, $\nabla_\mu(T^\mu_\nu + U^\mu_\nu) = 0$. Assuming that energy conservation holds independently for the conventional fluids, $\nabla_\mu T^\mu_\nu = 0$, (see ref [21] for the strengths and weaknesses of such an assumption) one then obtains the following two consistency equations:

$$\frac{2}{\mathcal{H}}\dot{A} + 2A - \frac{2k}{\mathcal{H}}B + C = 0; \quad (8)$$

$$6\dot{B} + kC + 12\mathcal{H}B - kD = 0. \quad (9)$$

Here we have defined $\mathcal{H} \equiv \dot{a}/a$ and dropped the arguments of the functions A to D .

Eqs. 3 to 6, together with Eqs. 8 and 9, show that two general functions of space and time would be enough to parametrize a wide range of modified theories of gravity. This method of course does not provide a test for any specific modified theory of gravity. However, given the current prejudice that GR is the true theory of gravity at low energies (eg, see Ref. [27] for a discussion), the main question is whether or not cosmological data can distinguish between GR and any other generic theory of modified gravity. Clearly, if we found evidence for deviations from GR, then we would have a parametric way of constraining the space of allowed models, and hence hone in on the correct theory.

3. CONNECTION WITH OTHER METHODS OF PARAMETRIZATION

The parameterization described in the previous section has a very close connection in practice with the PPF method explained in Ref. [21]. The most important differences are that we have grouped a number of different parameters into a single parameter, and have

used the synchronous gauge in Eqs. 3 to 6. The parameters defined in Sec. 2 are related to the PPF parameters according to:

$$A(k, \tau) = A_0 \hat{\Phi} + F_0 \hat{\Gamma} + \alpha_0 \hat{\chi} + \frac{\alpha_1}{k} \dot{\hat{\chi}}, \quad (10)$$

$$B(k, \tau) = B_0 \hat{\Phi} + I_0 \hat{\Gamma} + \beta_0 \hat{\chi} + \frac{\beta_1}{k} \dot{\hat{\chi}}, \quad (11)$$

$$C(k, \tau) = C_0 \hat{\Phi} + \frac{C_1}{k} \dot{\hat{\Phi}} + J_0 \hat{\Gamma} + \frac{J_1}{k} \dot{\hat{\Gamma}} + \gamma_0 \hat{\chi} + \frac{\gamma_1}{k} \dot{\hat{\chi}} + \frac{\gamma_2}{k^2} \ddot{\hat{\chi}}, \quad (12)$$

$$D(k, \tau) = D_0 \hat{\Phi} + \frac{D_1}{k} \dot{\hat{\Phi}} + K_0 \hat{\Gamma} + \frac{K_1}{k} \dot{\hat{\Gamma}} + \epsilon_0 \hat{\chi} + \frac{\epsilon_1}{k} \dot{\hat{\chi}} + \frac{\epsilon_2}{k^2} \ddot{\hat{\chi}}. \quad (13)$$

While the authors of Ref. [21] have insisted on the modifications being gauge invariant, it is good to keep in mind that there is nothing special about the use of gauge invariant parameters, as is shown in Ref. [28]. The important issue is to track the degrees of freedom in the equations. There are originally four free functions for the spin-0 degrees of freedom in the metric, but the gauge freedom can be used to set two of them to zero. Using only two gauge invariant functions instead of four, means that the gauge freedom has been implicitly used somewhere to omit the redundant variables.

All 22 of the parameters on the right hand side of Eqs. 10 to 13 are in fact two-dimensional functions of the wave number, k , and time. A hat on a function means that it is a gauge invariant quantity. $\hat{\chi}$ is the gauge invariant form of any extra degree of freedom that can appear, for example, in a scalar-tensor theory, or in an $f(R)$ theory as a result of a number of conformal transformations (see section D.2 of Ref. [21] for further explanation). $\hat{\Phi}$ and $\hat{\Gamma}$ are related to the synchronous gauge metric perturbations through:

$$\hat{\Phi} = \eta - \frac{\mathcal{H}}{2k^2}(\dot{\eta} + 6\dot{\eta}); \quad (14)$$

$$\hat{\Psi} = \frac{1}{2k^2}(\ddot{\eta} + 6\ddot{\eta} + \mathcal{H}(\dot{\eta} + 6\dot{\eta})); \quad (15)$$

$$\hat{\Gamma} = \frac{1}{k}(\dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi}). \quad (16)$$

One needs to add more parameters to the right hand side of Eqs. 10 to 13 if there is more than one extra degree of freedom, or if the equations of motion of the theory are higher than second order and the theory cannot be conformally transformed into a second order theory. The reason this many parameters was introduced in Ref. [21] is that there is a direct connection between these parameters and the Lagrangians of a number of specific theories, like the Horava-Lifshitz, scalar-tensor or Einstein Aether theories. Therefore, in principle, constraining these parameters is equivalent to constraining the theory space of those Lagrangians.

However, there are a number of issues that may encourage one to consider alternatives to the PPF approach for

parameterizing modifications to gravity. First of all, it is practically impossible to run a Markov chain code for 22 two-dimensional functions. One can reduce the number of free functions to perhaps 15 using Eqs. 8 and 9, but there is still a lot of freedom in the problem. The second reason is that the whole power of the PPF method lies in distinguishing among a number of classically modified theories of gravity that are mostly proven to be either theoretically inconsistent, like the Horava-Lifshitz theory [9], or already ruled out observationally, like TeVeS (at least for explaining away dark matter) [29]. While it is certainly important and useful to check the GR predictions with the new coming data sets, it does not seem reasonable at this stage to stick with the motivation of any specific theories. For the moment it therefore seems prudent to consider an even simpler approach, as we describe here.

There is another popular parametrization in the literature described fully in Ref. [30] and applied to the case of the CMB power spectrum in the same article. This second parametrization is best described in the conformal Newtonian gauge, via the following metric:

$$ds^2 = a^2(\tau)[-(1 + 2\psi)d\tau^2 + (1 - 2\phi)\delta_{ij}dx^i dx^j]. \quad (17)$$

The modifying parameters, $\{\mu, \gamma\}$, are defined as the following:

$$k^2\psi = -\mu(k, a)4\pi Ga^2\{\bar{\rho}\Delta + 3(\bar{\rho} + \bar{p})\Sigma\}; \quad (18)$$

$$k^2[\phi - \gamma(k, a)\psi] = \mu(k, a)12\pi Ga^2(\bar{\rho} + \bar{p})\Sigma. \quad (19)$$

Here $\Delta = \delta\rho + 3\frac{\mathcal{H}}{k}(1 + \bar{p}/\bar{\rho})V$, and all of the matter perturbation quantities are in the Newtonian gauge.

In order to see the connection between this method of parametrization and the one described in the previous section, through Eqs. 3 to 6, one needs to use the modified equations of motion in the Newtonian gauge:

$$k^2\phi + 3\mathcal{H}(\dot{\phi} + \mathcal{H}\psi) = -4\pi Ga^2\delta\rho + k^2 A_N; \quad (20)$$

$$k^2(\dot{\phi} + \mathcal{H}\psi) = 4\pi Ga^2(\bar{\rho} + \bar{p})kV + k^3 B_N; \quad (21)$$

$$k^2(\phi - \psi) = 12\pi Ga^2(\bar{\rho} + \bar{P})\Sigma + k^2 D_N. \quad (22)$$

The parameters $\{A_N, B_N, D_N\}$ are the modifying functions in the Newtonian gauge. These parameters are related to γ and μ via:

$$\alpha \equiv 1 - \mu, \quad (23)$$

$$\beta \equiv \gamma - 1, \quad (24)$$

$$4\pi Ga^2\alpha\{\bar{\rho}\Delta + 3(\bar{\rho} + \bar{p})\Sigma\} = k^2(A_N - 3\frac{\mathcal{H}}{k}B_N + D_N), \quad (25)$$

$$\beta\psi = 12\pi Ga^2\alpha\Sigma + k^2 D_N, \quad (26)$$

where, one can choose between using the functions $\{A_N, B_N, C_N, D_N\}$, along with two constraint equations similar to the Eqs. 8 and 9, or using the two parameters γ and μ and trying to remain consistent in the equations of motion.

It is argued in Ref. [31] that the $\{\gamma, \mu\}$ parametrization is not capable of parameterizing second order theories in the case of an unmodified background and no extra fields. To show this they use the fact that, in the absence of extra fields, all of the greek coefficients in Eqs. 10 to 13, i.e. $\{\alpha_0, \dots, \epsilon_2\}$, have to be zero. Furthermore, they argue that in the case of second order theories, F_0 and I_0 have to be zero, and therefore the constraint equations 8 and 9 show that J_0 and K_0 are zero as well. After all of this, one can see that equation 22 can be written as the following in this special case:

$$k^2(\phi - \psi) = 12\pi G a^2(\bar{\rho} + \bar{P})\Sigma + k^2(D_0\phi + \frac{D_1}{k}\dot{\phi}). \quad (27)$$

The authors then show that the absence of a term proportional to the metric derivative will lead to an inconsistency. This conclusion is valid only if one assumes that that β in Eq. 26 is a function of background quantities, which usually is not the case. Otherwise, one can use Eq. 26 to define β :

$$\beta \equiv \frac{12\pi G a^2 \alpha \Sigma + k^2(D_0\phi + \frac{D_1}{k}\dot{\phi})}{\psi}, \quad (28)$$

leaving no ambiguity or inconsistency.¹

It is also claimed in Ref. [32] that the $\{\gamma, \mu\}$ parametrization becomes ambiguous on large scales, while none of these shortcomings apply to the PPF method. However, these criticisms do not seem legitimate, since, as was shown in this section, there is a direct connection between $\{\gamma, \mu\}$, and the PPF parameters.² For any given set of functions for the PPF method, one can find a corresponding set of functions $\{\gamma, \mu\}$, using Eqs. 10 to 13 and 26, that will produce the exact same result for any observable quantity. One only needs to ensure the use of consistent equations while modifying gravity through codes such as CAMB.

Although we believe that there is no ambiguity in the $\{\gamma, \mu\}$ parameterization, we also believe that our $\{A, B, C, D\}$ parameterization can be implemented much more easily in codes such as CAMB and CosmoMC. Furthermore, there is a potential problem for the $\{\gamma, \mu\}$ parametrization on the small scales that enter the horizon during the radiation domination era. The metric perturbation ψ will oscillate around zero a couple of times for these scales and that makes the γ function blind to any modification at those instants of time. This also has the potential to lead to numerical instabilities.

4. NUMERICAL CALCULATION

In this section we will constrain the parameterization described in Sec. 2 using the CMB anisotropy power spectra. We will describe the effects of the modifying parameters on the CMB and show the results of numerical calculations from CAMB and CosmoMC.

4.1. Effects of A and B on the CMB power spectra

Before showing numerical results, we first describe some of the physical effects of having non-zero values of A or B . So far we have not placed any constraints on these quantities, which are in general functions of both space and time. There are some effects that can be explicitly seen from the equations of motion and energy conservation. For example, a positive A enhances the pressure perturbation and anisotropic stress, while reducing the density perturbation. On the other hand, a positive B will enhance the momentum perturbation and reduce the pressure perturbation and anisotropic stress.

There are also some other effects that need a little more algebra to see, and we now discuss three such effects:

Neutrino moments:

The neutrinos' zeroth and second moments, $\{F_{\nu 0}, F_{\nu 2}\}$, are coupled to the modifying gravity terms according to the Boltzmann equations [26] and Eq. 3 to 6:

$$\dot{F}_{\nu 0} = -k F_{\nu 1} - \frac{2}{3}\dot{h}; \quad (29)$$

$$\dot{F}_{\nu 2} = \frac{2}{5}k F_{\nu 1} - \frac{3}{5}k F_{\nu 3} + \frac{4}{15}(\dot{h} + 6\dot{\eta}). \quad (30)$$

Here \dot{h} is modified according to Eq. 3, and the term $\dot{h} + 6\dot{\eta}$ is coupled to A and B through equations 3 and 4:

$$\begin{aligned} \dot{h} + 6\dot{\eta} = & \frac{2k^2\eta + 8\pi G a^2\delta\rho}{\mathcal{H}} + 24\pi G a^2(\bar{\rho} + \bar{p})\frac{V}{k} \\ & - \frac{2k^2 A}{\mathcal{H}} + 6k B. \end{aligned} \quad (31)$$

Therefore, modified gravity can enhance the neutrino second moment.

Photon moments:

While the same thing is valid for the photons' second moment after decoupling, the situation is different during the tight coupling regime. The Thomson scattering rate is so high in the tight coupling era that this makes the second moment insensitive to gravity. In other words, the electromagnetic force is so strong that it does not let the photons feel gravity.

ISW effect:

The integrated Sachs-Wolfe (ISW) [33] effect is proportional to $\dot{\phi} + \dot{\psi}$ in the Newtonian gauge. In the synchronous gauge this is equal to

$$ISW \propto \dot{\phi} + \dot{\psi} = \frac{\ddot{h} + 6\ddot{\eta}}{2k^2} + \dot{\eta}. \quad (32)$$

¹ Note that this might be troublesome if ψ goes to zero at some moments of time. This happens for the scales that enter the horizon during radiation domination.

² In particular there is nothing wrong with the $\{\gamma, \mu\}$ parametrization on large scales, since ψ is certainly always non zero.

Here $\dot{\eta}$ is modified according to Eq. 4 and, therefore, a subtle change in the function $B(k, \tau)$, can have a considerable influence on the ISW effect. Fig. 1 shows the effects of a constant non-zero A and B on the ISW effect. For the case of a constant non-zero B , the ISW effect is always present, since the time derivative of the potential is constantly sourced by this function. This will result in more power on all scales, including the tail of the CMB power spectrum (see Fig. 2). Fig. 2 shows that if a non-zero B is favoured by CMB data, it is mostly due to the large l 's, ($l > 1500$), and it is due to its anti-damping behaviour.

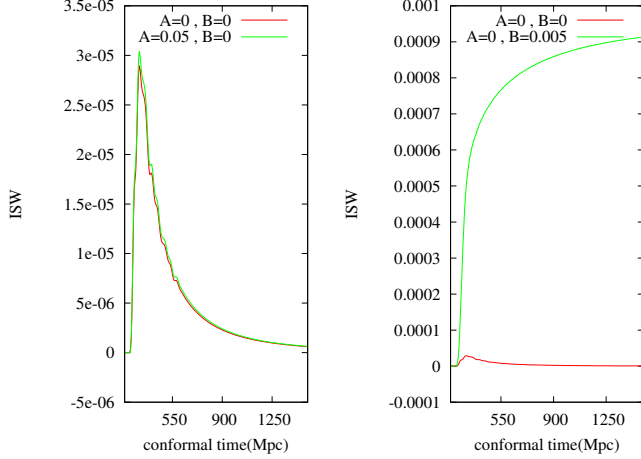


FIG. 1: The effects of a constant, non-zero A and B on the ISW effect. This plot shows the ISW effect for a scale of $k = 0.21 \text{Mpc}^{-1}$.

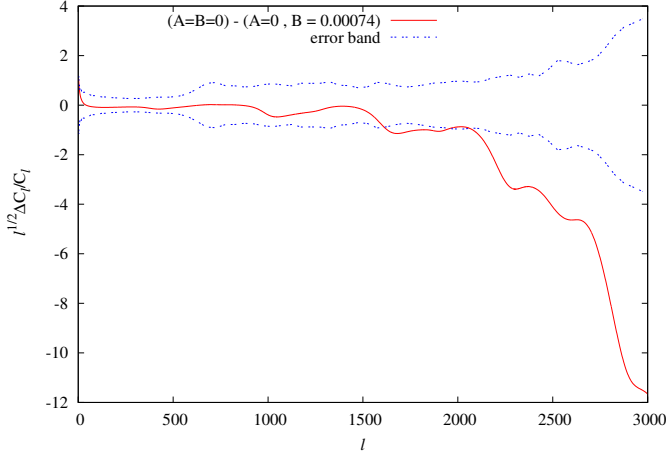


FIG. 2: Effects of a constant B on the CMB power spectra. The plot shows the difference in power for the case of zero B minus the best fit non-zero B , using *WMAP9* and *SPT12* data, while keeping all the rest of the parameters the same. The error band plotted is based on the reported error on the binned CMB power spectra from the *WMAP9* [23] and *SPT12* [24] groups.

Fig. 3 shows the CMB power spectra for the case of a constant but non-zero A and B . Note how a constant non-zero B brings the tail of the spectrum up. One might also point to the degeneracy of A and the initial amplitude mostly by looking at the height of the peaks.

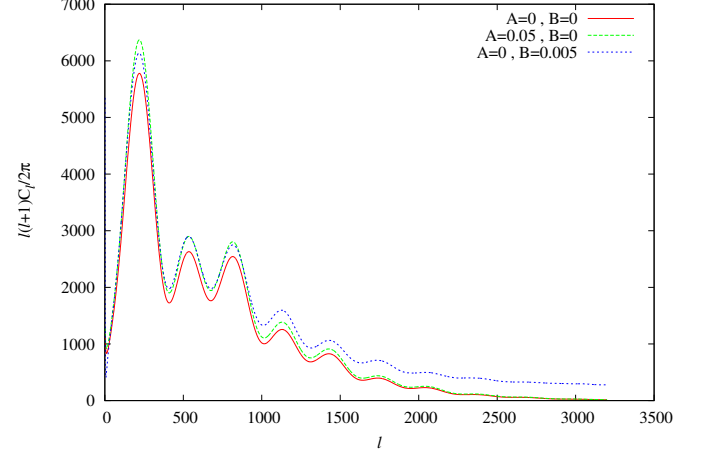


FIG. 3: The effects of a constant, non-zero A and B on the CMB power spectra. One can see that the two parameters have quite different effects.

Matter overdensity:

The Boltzmann equation for cold dark matter overdensity in the synchronous gauge reads [26]:

$$\dot{\delta}_{\text{CDM}} \equiv \left(\frac{\delta \rho_{\text{CDM}}}{\rho_{\text{CDM}}} \right)' = -\frac{1}{2} \dot{h}. \quad (33)$$

Using Eqs. 3, 4, and the Friedmann equation, $\mathcal{H}^2 = \frac{8\pi G a^2}{3} \bar{\rho}$, and assuming a matter dominated Universe with no baryons, one gets the following equation for the cold dark matter overdensity:

$$\begin{aligned} \mathcal{H} \dot{\delta}_{\text{CDM}} &= -\frac{3\mathcal{H}^2}{2} \delta_{\text{CDM}} + k^2 A - k^2 \eta \Rightarrow \\ \dot{\delta}_{\text{CDM}} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}} + \frac{3}{2} \mathcal{H} \right) \delta_{\text{CDM}} + 3\dot{\mathcal{H}} \delta_{\text{CDM}} &= \\ -\frac{k^3}{\mathcal{H}} B + \frac{k^2}{\mathcal{H}} \dot{A}. \end{aligned} \quad (34)$$

The above equation clearly shows the role of A and B as driving forces for matter overdensity. The k^3 prefactor makes the first term on the right hand side dominant on the small scales and will therefore have a significant effect on the matter fluctuation amplitude at late times. Matter power spectrum will therefore put strong constraints on modified gravity models.

4.2. Markov Chain constraints on A and B

Since $A(k, \tau)$ and $B(k, \tau)$ are free functions, we need to choose some simple cases to investigate. We choose here

to focus on the simple cases of A and B being separate constants (i.e. independent of both scale and time). We do not claim that this is in any sense a preferred choice — we simply have to pick something tractable. With better data one can imagine constraining a larger set of parameters, for example describing A and B as piecewise constants.

We used CosmoMC to constrain constant A and B , together with the *WMAP*-9 [23] and SPT12 [24] CMB data. The amplitudes of the foregrounds were added as additional parameters and were marginalized over for the case of SPT12. The resulting constraints and distributions are shown in Fig. 4.

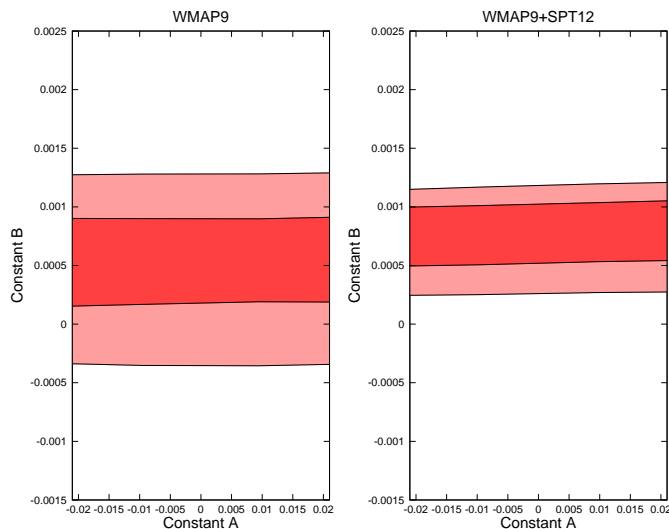


FIG. 4: The 68 and 95 percent contours of the constants A and B using WMAP9-year data alone (left) and WMAP9 + SPT12 (right).

There is a strong (anti-)correlation between A and the initial amplitude of the scalar perturbations, and therefore the constraints on A are quite broad. Two-dimensional contour plots of A versus A_s , the initial amplitude of scalar curvature perturbations, are shown in Fig. 5.

The mean of the likelihood and 68% confidence interval for the six cosmological parameters together with A and B are tabulated in Table I. Notice that while *WMAP*9 is fairly consistent with the conventional Λ CDM model, there is a 3σ preference for non-zero B when one adds the SPT12 data.

5. DISCUSSION

Since a constant A is essentially degenerate with the initial amplitude of the primordial fluctuations, the CMB cannot constrain this parameter. On the other hand, B seems to be fairly well constrained by the data. However,

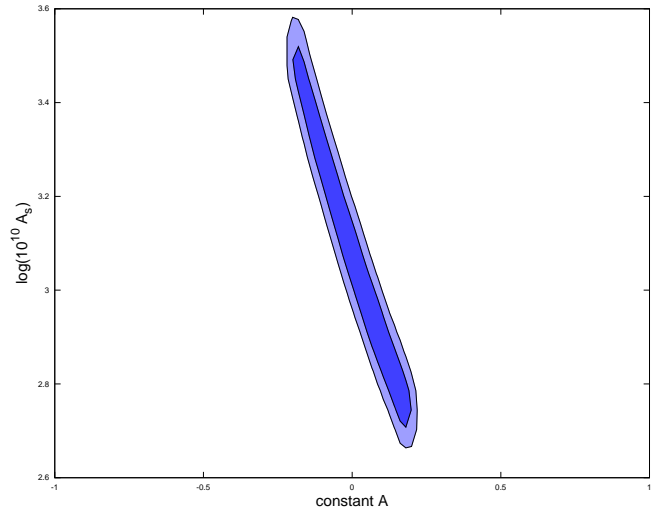


FIG. 5: The strong anti-correlation between parameter A and the initial amplitude, A_s makes the constraints on any one of these two parameters weaker.

if B was an oscillating function of time, changing sign from time to time, its total effect on the CMB power spectra would become weaker and the constraints would be broader. According to Eq. 4, a constant B will change η monotonically, while the effect of an oscillating B will partially cancel some of the time.

By comparing the results of the MCMC analysis for *WMAP* and SPT, it seems like a non-zero B is somewhat favoured by data. Essentially what is happening here is that a mild tension between *WMAP* and SPT is resolved by adding an appropriate degree of freedom. One can see from Fig. 3 that crudely, a constant B changes the normalization of the large-scale vs. small-scale power spectrum. However, it will take significantly better data to determine whether the improved fit genuinely looks like a modification of gravity, or has a more mundane explanation. In particular, as was mentioned before, the amplitudes of the foreground power spectra were considered unknown parameters and were fitted and marginalized over to obtain the distributions for A and B here. Since B is slightly correlated with all of these amplitudes (and mostly with the point source shot-noise amplitude), then upcoming data from the *Planck* satellite certainly will be helpful.

Acknowledgments

We acknowledge many detailed and helpful discussions with our colleague James Zibin. We also had very useful conversations about this and related work with Levon Pogosian. This research was supported by the Natural Sciences and Engineering Research Council of Canada.

Parameter	WMAP9	WMAP9+SPT12
$100 \Omega_b h^2$	2.22 ± 0.05	2.10 ± 0.03
$\Omega_{DM} h^2$	0.118 ± 0.005	0.122 ± 0.005
100θ	1.038 ± 0.002	1.040 ± 0.001
τ	0.086 ± 0.014	0.076 ± 0.012
$\log(10^{10} A_s)$	3.1 ± 0.2	3.1 ± 0.2
n_s	0.96 ± 0.01	0.93 ± 0.01
$100A$	1 ± 10	0.02 ± 10
$1000B$	0.44 ± 0.41	0.74 ± 0.24

TABLE I: The mean likelihood values together with the 68% confidence interval for the usual six cosmological parameters, together with constant A and B .

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